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Historical Development of Aircraft Flutter

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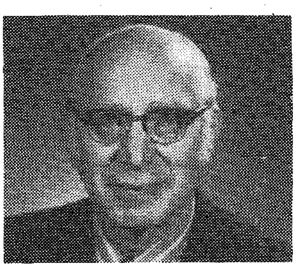
Introduction

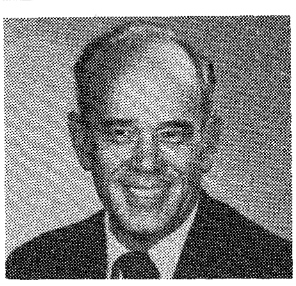
AEROELASTICITY, and in particular flutter, has influenced the evolution of aircraft since the earliest days of flight. This paper presents a glimpse of problems arising in these areas and how they were attacked by aviation's pioneers and their successors up to about the mid-1950s. The emphasis is on tracing some conceptual developments relating to the understanding and prevention of flutter including some lessons learned along the way.

Because it must be light, an airplane necessarily deforms appreciably under load. Such deformations change the distribution of the aerodynamic load, which in turn changes the deformations; the interacting feedback process may lead to *flutter*, a self-excited oscillation, often destructive, wherein energy is absorbed from the airstream. Flutter is a complex phenomenon that must in general be completely eliminated by design or prevented from occurring within the flight envelope. The initiation of flutter depends directly on the stiffness, and only indirectly on the strength of an airplane, analogous to depending on the slope of the lift curve rather than on the maximum lift. This implies that the airplane must be treated not as a rigid body but as an elastic structure. Despite the fact that the subject is an old one, this requires for a modern airplane a large effort in many areas, including ground vibration testing, use of dynamically scaled wind-tunnel models, theoretical analysis, and flight flutter testing. The aim of this paper is to give a short history of aircraft flutter, with emphasis on the conceptual developments, from the early days of flight to about the mid-1950s.

Work in flutter has been (and is being) pursued in many countries. As in nearly all fields, new ideas and developments in flutter have occurred similarly and almost simultaneously in diverse places in the world, so that exact assignment of priorities is often in doubt. Moreover, a definitive historical account would require several volumes; yet we hope to survey some of the main developments in a proper historical light, and in a way that the lessons learned may be currently useful. It is recognized that detailed documentation of flutter troubles has nearly always been hampered by proprietary conditions and by a reluctance of manufacturers to expose such problems.

From our present perspective, flutter is included in the broader term aeroelasticity, the study of the static and dynamic response of an elastic airplane. Since flutter involves the problems of interaction of aerodynamics and structural deformation, including inertial effects, at subcritical as well as at critical speeds, it really involves all aspects of aeroelasticity. In a broad sense, aeroelasticity is at work in natural phenomena such as in the motion of insects, fish, and birds (biofluid-dynamics). In man's handiwork, aeroelastic problems of windmills were solved empirically four centuries ago in Holland with the moving of the front spars of the blades from about the midchord to the quarter-chord position (see the article by Jan Drees in list of Survey Papers). We now recognize that some 19th century bridges were torsionally weak and collapsed from aeroelastic effects, as did the Tacoma Narrows Bridge in spectacular fashion in 1940. Other aeroelastic wind-structure interaction pervades civil





After a long career as research scientist and manager, I. E. Garrick retired from NACA/NASA in 1972. He remains active as Distinguished Research Associate of Langley. He is the author of numerous publications in areas of aerodynamics, aeroelasticity, and aeromechanics, has served on many advisory councils, and has lectured extensively. Mr. Garrick served for a year as the second Hunsaker Professor of Aeronautical Engineering at MIT. His 1976 AIAA von Kármán Lecture dealt with the topic: "Aeroelasticity—Frontiers and Beyond." Among his awards are the NASA Exceptional Service Award, the Langley Scientific Achievement Award, and the AIAA Sylvanus A. Reed Award. He is a Fellow of the AIAA.

Wilmer H. Reed III from 1972 to 1980 was Head of the Aeroelasticity Branch which operates the Langley Transonic Dynamics Tunnel, a national facility dedicated exclusively to research and development in the field of aeroelasticity. Notably, this facility is used by the United States in development of practically all its military aircraft, commercial transports, and launch vehicles. At present, Mr. Reed is Chief Scientist, Loads and Aeroelasticity Division, NASA Langley Research Center. He joined NACA/NASA in 1948 and holds Bachelor and Master degrees in Aeronautical Engineering from Auburn University and the University of Virginia. His professional experience includes research in aeroelasticity, wind loads on structures, damping devices, and wind tunnel testing techniques. He received the NASA Exceptional Service Medal in 1979, holds several U.S. Patents, is a member of Tau Beta Pi, and an Associate Fellow of the AIAA.

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engineering. The elastic response of an airplane to rough air (gusts or turbulence) is an important aeroelastic problem requiring separate study and documentation.

As the phenomena and concepts have unfolded, aeroelasticity, and flutter in particular, have been the subjects of many survey papers throughout the years. These papers often furnish valuable assessments of the state-of-the-art, give interesting bits of the history, and also furnish numerous useful references. As it is feasible to refer to only a small fraction of these references individually, we have included a list of such survey papers separate from quoted references. In particular, we may refer the reader to the outstanding survey papers of A. R. Collar, which emphasize the British developments, the most recent of which, "The First Fifty Years of Aeroelasticity," came to our attention during the writing of this paper. It was Collar's "Aeroelastic Triangle" (1947) that showed graphically that flutter embraced all aspects of aeroelasticity.

The Early Years, 1903-1919

The Wright Brothers

In their historic flight, Orville and Wilbur Wright made beneficial use of aeroelastic effects for roll control of their biplane by the use of wing warping in place of ailerons. They also were aware of the adverse aeroelastic effect of the loss of thrust of a propeller, due to twisting of the blades, by their experiments on the performance of thin propellers having broad blades. They found that the propeller tip under high thrust loads twisted to partially unload itself. We quote from *The Papers of Wilbur and Orville Wright*[1]:

> Orville Wright, in later life, explained the nature and function of the "little jokers" to Fred C. Kelley, who states in his book, *The Wright Brothers,* "After the Wrights had made the blades of their propellers much wider and thinner than the original ones, they discovered that the performance of the propellers in flight did not agree closely with their calculations, as in the earlier propellers. They could see only one reason for this, and that was that the propeller blades twisted from their normal shape under pressure in flight. To find out quickly if this was the real reason, they fastened to each blade a small surface, like an elevator, out behind the blades, set at an angle to balance the pressures that were distorting the blades. They called the surfaces 'little jokers.' When they found that the 'little jokers' cured the trouble, they dispensed with them and began to give the blades a backward sweep which served the same purpose."

S. P. Langley and His Aerodrome

On December 8, 1903, only nine days before the Wright brothers' flight at Kitty Hawk, Professor Samuel P. Langley of the Smithsonian Institute failed, for the second time, in an attempted launch of his powered flying machine from the Potomac River houseboat (Fig. 1). In both instances, Langley's tandem monoplane plunged into the Potomac as a result of structural failures encountered during the catapulted launch. The failure of the first attempt has been attributed to the front-wing guy post being caught on the launch mechanism and not releasing as planned. The cause of failure in the second flight, which involved collapse of the rear wing and tail, is less certain.

It has been conjectured that aeroelasticity may have played a major role in the second failure. G.T.R. Hill[2] suggested that the failure was the result of insufficient wing-tip stiffness resulting in wing torsional divergence, a nonoscillatory aeroelastic instability that may be regarded as flutter at zero frequency. Hill's argument is bolstered by a qualitative, but highly perceptive, discussion given in 1913 by Griffith

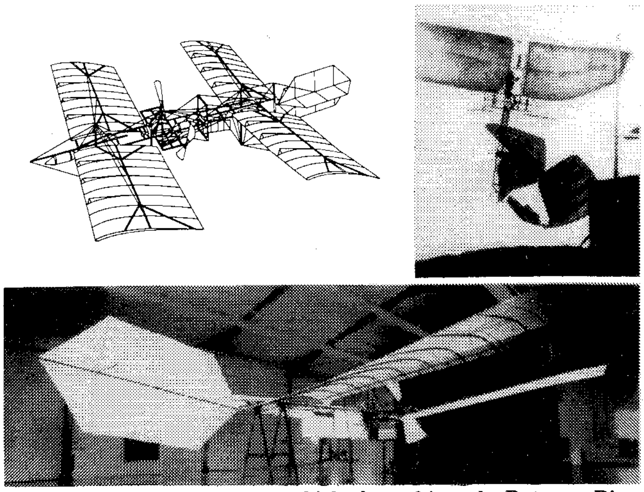

Fig. 1 Langley's aerodrome, which plunged into the Potomac River in 1903.

Brewer[3] on the collapse of monoplane wings. Brewer notes in his one-page article in *Flight Magazine* that a rash of monoplane wings with stays had had "accidents in which the wings break *downward*" and he remarks that "the greater the span the more readily will the wing tips be twisted"; and that the movement of the center of pressure with speed could put it behind the attachment point of the rear stay, resulting in sudden wing flip.

Professor Collar (see Survey Papers, 1958, 1978) has stated that part of the speculation about Langley's airplane disaster is based on the circumstances that some years after Langley's death the original Langley machine was removed from the Smithsonian National Museum, modified, and flown successfully at Hammondsport, N.Y. These modifications, which involved substantial changes in the wing structure and trussing, so strengthened and stiffened the original structure as to significantly reduce the probability of aeroelastic failure. This led in later years to a long controversy (see Brewer[4] and Pritchard[5]) about whether the original craft was *capable* of manned flight, preceding the Wright brothers in this aspect. Brewer showed from photographs taken during the first launching that the wings were twisting excessively. Professor Collar summarized the speculations with the remark, "It seems that but for aeroelasticity Langley might have displaced the Wright brothers from their place in history." After the Hammondsport trials, the machine was reconstructed from the remaining components to its original configuration and returned to the Smithsonian Institute in Washington.

Recently, the authors were given a unique opportunity to shed some new light on the role aeroelasticity may have played relative to the collapse of the Langley machine in 1903. At the invitation and encouragement of Mr. Melvin Zisfein, Deputy Director of the National Air and Space Museum and a former prominent aeroelastician, the authors made some relevant measurements on the restored 1903 Langley machine, which had recently undergone refurbishment to its original condition by the museum's restoration facility in Suitland, Md. By applying a vertical point load to the wing at various chordwise locations the so-called "elastic axis," defined by the property that a load applied at the elastic axis does not cause twist, was found to lie along the quarter-chord line.* With the elastic axis location this far forward, it appears unlikely that the wing failed as a result of static aeroelastic divergence, as had been suggested. Nevertheless, because the highly cambered wing produces a downward twisting moment

*These tests were conducted by Wilmer Reed III, Rodney Ricketts, and Robert Doggett of the Langley Research Center with the assistance of Dr. Harold Walco and Joseph Fichera of the Smithsonian.

and because of the overall lack of structural rigidity, especially torsional rigidity of the wing and fuselage, it still remains highly probable that the collapse of the machine during launch can be attributed to aeroelastic effects, such as overload due to elastic deformations during launch in an untrimmed condition.

The success of the Wright biplane and the failure of the Langley monoplane may have influenced early aircraft designers' preference towards biplanes. Undoubtedly, the structural justification for the biplane vs the externally braced monoplane comes from the inherent wing stiffness readily achieved on biplanes by means of interplane struts and cross bracing (Fig. 2).

Lanchester and Bairstow—The First Documented Flutter Study

The first major development in flutter was accomplished by the preeminent British engineer and scientist F. W. Lanchester[6] during World War I in troubleshooting why the Handley Page 0/400 biplane bomber had experienced violent antisymmetric oscillations of the fuselage and tail. The aircraft's right and left elevators were essentially independent, being connected to the stick flexibly by separate cable runs. Lanchester recognized, and described in a masterful text of only three pages, two important concepts: 1) that the oscillations were not the result of resonance induced by vibratory sources but were *self-excited* and 2) that increase of the torsional stiffness of the elevators by means of a carry-through torque tube could eliminate the problem. Another epidemic of tail flutter resulting in pilot fatalities was experienced only a year later by the de Havilland DH-9 airplane. The cure was identical to that suggested by Lanchester; the torsionally stiff connection between the elevators has remained ever since an important design feature (Fig. 3).

In Lanchester's investigation of the Handley-Page airplane Leonard Bairstow provided analytical backup in the investigation of the Handley-Page airplane. A resulting paper by Bairstow and Fage[7] is probably the first theoretical flutter analysis. They investigated binary flutter consisting of the two degrees of freedom: twisting of the fuselage body and motion of the elevators about their hinges, as described in Fig. 4. The dynamical equations of motion were patterned after small disturbance methods of the analysis of stability of rigid-body aircraft that were under study from 1903, and summarized in the classic book of G. H. Bryan.[8] Bairstow had written many papers on stability using Bryan's methods. Aerodynamic coefficients for stability analysis have been termed *derivatives* since they are applied for small deviations from equilibrium flight paths. The aerodynamic derivatives of Bairstow and Fage were constant coefficients multiplied by an exponential time factor, and referred to as quasistationary constants. The two equilibrium equations of motion were homogeneous, so that the determinant of their coefficients gave a quartic polynomial for determining the roots (eigenvalues) and free modes (eigenmodes). By examination of Routh's criteria,[9] obtained from the coefficients of the polynomial, one could determine, without solving for roots, whether any instability, oscillatory or divergent, existed. Bairstow and Fage had to make reasonable estimates for the quasistationary aerodynamic constants, but the investigation fully confirmed Lanchester's conclusions, and set a pattern for the extensive British work that was to evolve a decade later.

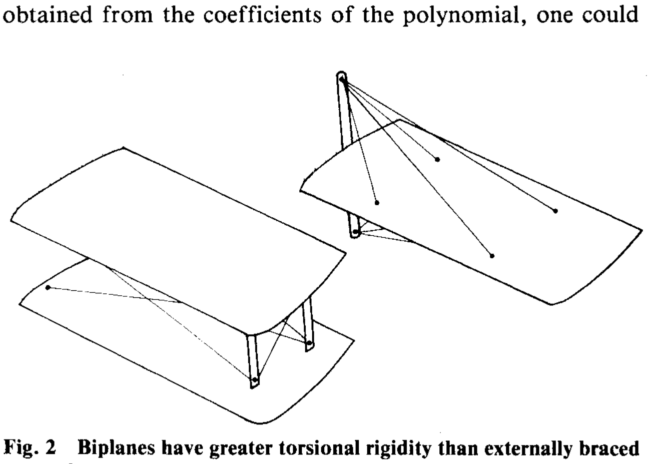

Fig. 2 Biplanes have greater torsional rigidity than externally braced monoplanes.

German Fighters, Anthony Fokker—Torsional Divergence

On the German side in World War I, many fatal structural failures on two fighter designs were attributed to aeroelastic static divergence problems. The German Albatros D-III, a biplane patterned after the French Nieuport 17, had a narrow single-spar lower wing connected by a V-strut to a large upper wing (see Fig. 5). Because the lower wing spar was positioned too far aft and the V-strut contributed no torsional stiffening to it, the wing tended to twist and wrench loose in high-speed dives. German ace Manfred von Richthofen, "The Red Baron," was among the lucky few who were able to land safely after dangerous cracks developed in the lower-wing spar during combat.

Near the end of the war, Fokker introduced the Fokker D-VIII, a cantilever parasol monoplane which was rushed into production because of its superior performance. Almost immediately, however, serious problems were encountered, as described by Bisplinghoff, Ashley, and Halfman (see Books) "The D-VIII was not in combat more than a few days before wing failures repeatedly occurred in high-speed dives. Since the best pilots and squadrons were receiving them first, it appeared possible that the flower of the German Air Corps would be wiped out. After a period in which the Army engineers and the Fokker Company each tried to place the responsibility on the other, the Army conducted static strength tests on half a dozen wings and found them suf-

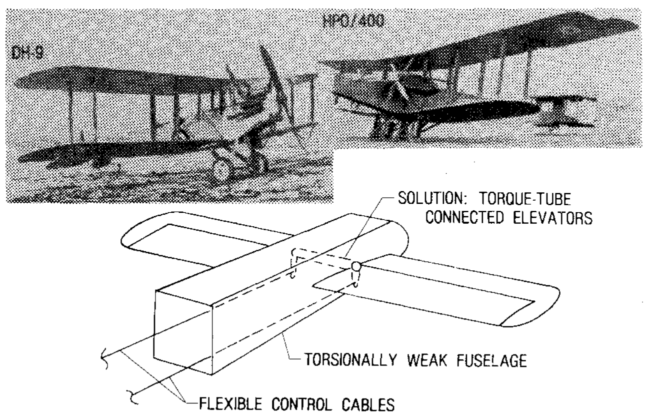

Fig. 3 Lanchester's solution to tail-plane flutter (1916).

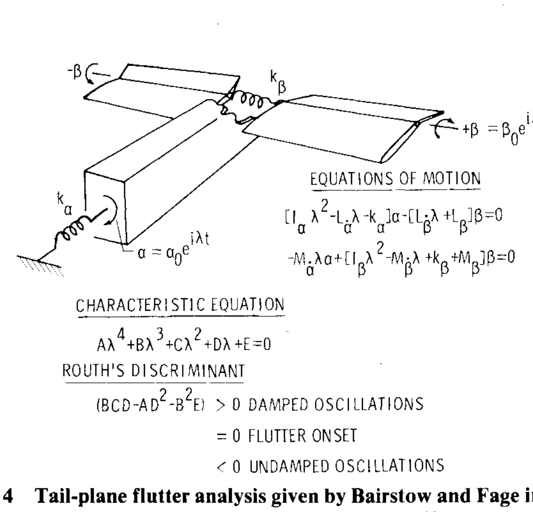

Fig. 4 Tail-plane flutter analysis given by Bairstow and Fage in 1916. (The I terms are moments of inertia, k terms are stiffnesses, and L and M terms are aerodynamic derivatives.)

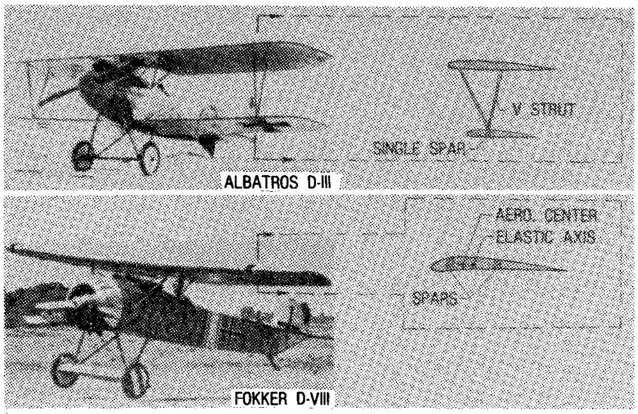

Fig. 5 Wing collapse of World War I German fighters caused by aeroelastic divergence.

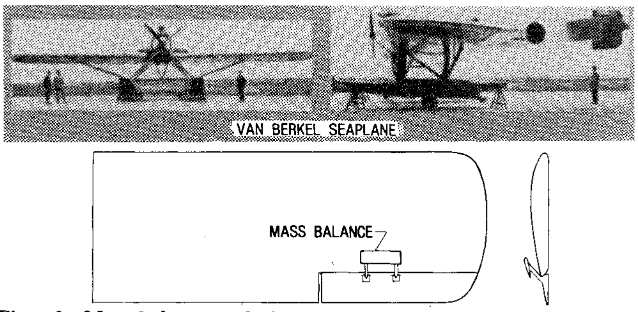

Fig. 6 Mass-balance solution of wing-aileron flutter of von Baumhauer and Koning (1923).

ficiently strong to support the required ultimate factor of 6." The only difference between the prototype wing, which had shown no structural deficiencies, and the production wing was a strengthening of the *rear* spar of the production wing. This strengthening had been ordered by the Army on the basis of regulations developed for wire-braced wings, which called for proportional strength in the rear spar and the front spar. Ironically, although made stronger, the production wing had unknowingly been made prone to aeroelastic divergence because of the shift of its elastic axis (Fig. 5). In Fokker's own words[10]: "I discovered (during the strength tests) that with increasing load the angle of incidence at the wing tips increased perceptibly. I did not remember having observed this action in the case of the original wings, as first designed by me. It suddenly dawned on me that this increasing angle of incidence was the cause of the wing collapse, as logically the load resulting from the air pressure in a steep dive would increase faster at the wing tips than in the middle, owing to the increased angle of incidence. It was the strengthening of the rear spar which had caused an uneven deflection along the wing under load.... The resulting torsion caused the wing to collapse under the strain of combat maneuvers." It is noteworthy that, as mentioned earlier, the Wright brothers had observed the related aeroelastic effect for thin-bladed propellers.

We mention here that in 1926, H. Reissner, author of many papers on aircraft structures, developed a detailed analysis of wing torsional divergence,[11] showing the importance of the relative locations of the aerodynamic center of pressure and of the elastic axis. This axis has been called the axis of twist, or in the British literature, the flexural axis. It is, as mentioned, defined by the property that a section loaded vertically at the flexural axis does not twist, or reciprocally, that a moment applied about this axis does not cause bending. It is a useful concept primarily for beam-like wings of moderate to high aspect ratio. In general, for more complex structures the use of nodal lines of vibration modes, or influence coefficients is more appropriate.

Post World War I to About 1930

von Baumhauer and Koning—Mass Balance Concept

Shortly after World War I a major systematic study of flutter was undertaken in the Netherlands following severe aileron flutter of a van Berkel W.B. monoplane, a long-distance reconnaissance seaplane. An experimental and theoretical investigation undertaken by A. G. von Baumhauer and C. Koning[12] was published in 1923. They dealt mainly with the binary flutter of the wing in vertical bending combined with motion of the ailerons. The most significant result of their study was the recognition that mass balance of the aileron, or even partial mass balance, could eliminate the problem (Fig. 6). Thus the concept of *decoupling of interacting modes* to prevent flutter was emphasized. A letter to the authors from H. Bergh of the National Aerospace Laboratory (NLR) of the Netherlands rightly observes that the 1923 investigation already contained features of a modern flutter investigation: 1) analysis of the observed phenomenon, 2) derivation of the equations of motion, 3) determination of mass and stiffness properties, 4) measurement of aerodynamic derivatives, 5) stability calculations, 6) flutter measurements in a wind tunnel, 7) comparison of theoretical and experimental flutter results, 8) the special flutter remedy, the mass balance concept, 9) verification by wind tunnel experiments and flight tests.

However, a main shortcoming not improved for many years, was the use of the approximate and empirical quasistationary aerodynamic constants. One may add that in the tradition of this work the small Netherlands research group has been productive in flutter research to the present time.

British Experience and Research, 1925-1929

A year later in the 1924-1925 yearbook of the British Aeronautical Research Committee (ARC) Chairman R.T. Glazebrook wrote, "Of increasing importance is the problem of flutter which has been discussed with representatives of a number of firms; a preliminary theoretical attack has been made on the problem. It would appear that the subject may need a large amount of experimental inquiry before a complete solution is obtained. Information on the rigidity of wings is being collected by the Airworthiness Department of the Air Ministry, and a series of accidents associated with flutter is being investigated by the Accidents Subcommittee."

This farseeing statement is of interest for several reasons. The technical word flutter is introduced here as though it were a commonly used term. Yet it appears for the first time, and may have become familiar within the confines of committee discussions. The committee subsequently assigned responsibility for flutter research, for both simple model and full-scale work, to two organizations, the National Physical Laboratory (NPL) and the Royal Aeronautical Establishment (RAE). Shortly afterward it established a Flutter Subcommittee.

One of the first publications[13] of the ARC Accidents Subcommittee describes in 1925 five incidences of wing-aileron flutter on two similar single seater biplane designs, the Gloster Grebe and the Gloster Gamecock. Flutter of about 15 cycles/s is described as having the appearance of a "blurr" to the pilot and as a "hovering hawk" to a ground observer. The remedy chosen was to move the aileron interplane connecting strut close to the center of mass and to reduce the unbalanced area near the tip (Fig. 7). It is of special interest that the Subcommittee stated that "similar flutter experiences have been reported both in Holland and in the U.S."

After three years of intensive work, mainly described in unpublished documents, a remarkably comprehensive monograph by Frazer and Duncan[14] of the NPL was published in 1929, often referred to by British workers as "The Flutter Bible." It made use of simplified wind tunnel models to identify and study phenomena, gave well-considered cautiously detailed design recommendations,[15]

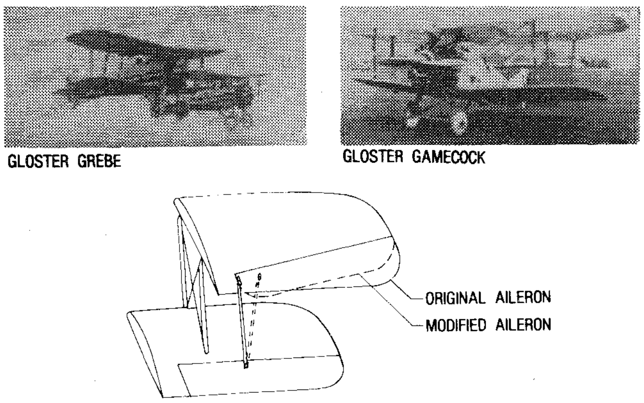

Fig. 7 Wing-aileron flutter led to intensive investigations by the British at the RAE and NPL (1925-1929).

and indicated broad programs required for measurement of aerodynamic derivatives. They introduced an important concept of "semirigid" modes which greatly simplifies the theoretical analysis by defining a dynamical degree-of-freedom as a motion in a given "shape of oscillation." In effect this concept enables the problem to be handled by ordinary differential equations rather than by much less tractable partial differential equations. The concept is directly related to ideas of Rayleigh in the use of assumed modes in treating ordinary vibration problems of conservative systems, where the cross-coupling coefficients are symmetrical. In flutter we are dealing with far more complicated systems (nonconservative, non-self-adjoint). It must be stated, however, that the aerodynamic basis of the work of Frazer and Duncan was not satisfactory, resting as it did on empirical aerodynamic constants which took no account of the interaction effect of the wake of shed vortices. Duncan himself remarks in the AGARD *Manual on Aeroelasticity* (see Survey Papers), "All the early purely theoretical work on flutter was marred by the inadequacy of the representation of the aerodynamic action."

Along with the work of Frazer and Duncan, which had used flutter models to study trends and validate theory, a companion publication by Perring[16] (1928) initiated the use of scaled models to determine the critical flutter speeds of an airplane prototype. The configuration selected for the study was the single seater biplane whose wing-aileron flutter encounters in flight had been well documented by the Accidents Investigation Subcommittee. Scaling laws developed in unpublished documents by McKinnon Wood and by Horace Lamb (1927) required that for dynamic similarity between a model and its full-scale counterpart there must be similarity in geometry, mass, and elastic distributions. Scaling parameters involving the effects of compressibility (Mach number), viscosity (Reynolds number), and gravity (Froude number) were considered in these studies to be unimportant and were therefore ignored. A one-third scale semispan model having the same mass density but with stiffnesses reduced to one-ninth that of full scale was tested in the RAE 7-ft wind tunnel. The flutter speeds and frequencies of the model correlated well with those observed on the full-scale machine. This study was one of the first to demonstrate the efficacy of the aeroelastically scaled wind-tunnel model as a means for predicting critical flutter speeds of a full-scale prototype.

Unsteady Aerodynamics in the 1920s

In 1918, Professor Prandtl in Göttingen assigned a thesis problem on airfoil theory to W. Ackerman, and later reassigned the unfinished work to W. Birnbaum when Ackerman was called into war service. Birnbaum published two important papers, the first of which gave in 1923 the classical vortex theory of the two-dimensional steady flow of thin airfoils (Max Munk had published his thin airfoil theory some months earlier, while H. Glauert gave an alternative formulation a year later). Birnbaum was able to extend his approach to the *harmonically oscillating* airfoil in uniform motion.[17] He made use of the concept of an oscillating vorticity distribution bound over the airfoil and free floating in the wake, the total circulation being zero by Kelvin's theorem of conservation of vorticity. By expressing the free vorticity in terms of the bound vorticity, he obtained an integral equation which yielded the pressure in terms of the known normal velocity at the airfoil surface. A series expansion for the pressure was introduced in terms of a nondimensional frequency, the *reduced* frequency ω (the frequency times the chord divided by the velocity), each term of which automatically satisfied the Kutta condition by the vanishing of the loading at the trailing edge. The numerical values, however, did not converge well beyond about $\omega > 0.10$.

A second basic approach to the theoretical problem of nonstationary flow supplementing the harmonic approach of Birnbaum, was given by H. Wagner in a doctoral thesis in 1925.[18] He studied the growth of vorticity in the wake and the growth of lift on an airfoil in two-dimensional flow following a sudden change of angle of attack, or a sudden acquisition of unit downwash. This type of response is now called indicial, after a terminology used in electric circuit analysis for the response to a Heaviside unit step type of excitation. Wagner's analysis made auxiliary use of the conformal mapping of the straight line into a circle, and led to an integral equation giving the growth of the free vorticity in the wake, from the solution of which the growth of lift followed. The integral equation satisfied the property that the Kutta trailing-edge condition should hold at each instant of time. The resulting function giving the growth of lift with distance traveled has been designated as Wagner's function $k_1(s)$.

In 1929, H. Glauert,[19] following Wagner's methods, treated the flat plate airfoil undergoing steady angular oscillations. He gave integral expressions for the lift and moment that were evaluated numerically, and were not subject to the convergence difficulties of Birnbaum. (Several years later, J. M. Burgers[20] showed that Glauert's integrals could be expressed by Bessel functions.) Glauert also called attention to the circumstance that the damping moment in pitching could change sign and that it indicated a mild type of single-degree-of-freedom flutter occurring at very low frequencies and for far-forward positions of the axis of rotation. Study of how this type of instability is affected by configuration and by Mach number was made many years later in several investigations.

In the very same year, H.G. Küssner,[21] published a basic paper on flutter, utilizing improvements on Birnbaum's method. He improved the numerical convergence to values of the reduced frequency $\omega \approx 1.0$, and applied the results, with

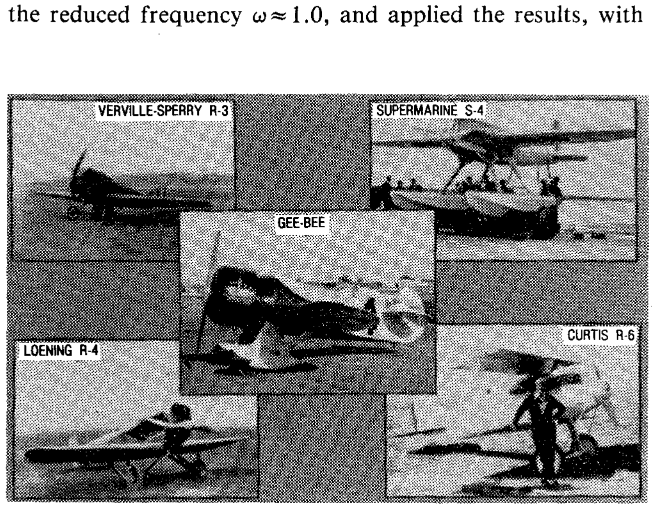

Fig. 8 Some air racers plagued by flutter (1922-1931).

the use of beam theory, to many examples involving bending and torsion including aileron motion. Significantly, he also investigated effects of hysteresis-type damping to represent material damping as affecting the flutter results. Küssner also indicated ground vibration methods aimed at checking the structural basis of the flutter analysis, for either model or full-scale structure, in the laboratory.

It seems appropriate to remark that in October 1980 both Wagner and Küssner, each in his eightieth year, were honored for their lifelong contributions to aeronautics by the awards of the prestigious Ludwig Prandtl Ring for the years 1980 and 1981.

During the period 1925-1929, several other prominent German workers used beam-rod concepts to investigate flutter, utilizing, however, quasisteady aerodynamics which neglected the trailing wake effects. Among these were H. Blasius (1925), B. Hesselbach (1927), and, notably, H. Blenk and F. Liebers (1927-1929).

Some Early United States Work

One of the earliest investigations of flutter in the United States (1927) was that of the horizontal tail oscillations, at about 6 cycles/s, of the Navy MO-1 airplane. After eliminating the wake of the main wing as the cause of the excitation, Zahm and Bear[22] made an analysis for flutter that followed closely the methods used in the Netherlands and England. The problem was traced to flutter involving the differential deflections of the two-spar system which produced a strong coupling between bending and torsion. Recommendations given for its avoidance included increased torsional stiffness and forward shift of the center of mass. Other introductory articles on flutter published in the 1927-1928 period were by J.S. Newell, by J.E. Younger, and by C.F. Greene.[23]

In 1927, some flutter work was started at the Massachusetts Institute of Technology (MIT) by Manfred Rauscher in a student thesis dealing with the use of models in the wind tunnel. Other student thesis work on models was initiated by G.W. Grady and F. MeVay. A published version by Rauscher[24] of this work, in German, described the use of models to attempt to verify the then current work of Blenk and Liebers, with the conclusion that the comparisons were unsatisfactory and much remained to be done.

It is appropriate to mention that a dozen years earlier at MIT, J.C. Hunsaker and E.B. Wilson had published some gust and stability studies in NACA Rept. No. 1, "Behavior of Airplanes in Gusts," showing by theory and experiment that the response of an airplane to given gusts, an aeroelastic problem, was sensitive to the stability limits of the airplane. Later, at the outset of World War II, Rauscher led the MIT Aeroelastic and Structures Research Laboratory in a program, wherein elaborate models aimed at dynamic similarity of typical military airplanes were constructed and tested. This laboratory had associated with it prominent leaders like R.L. Bisplinghoff, H. Ashley, R.L. Halfman, R. Laidlaw, Rene Miller, Eric Reissner, E. Mollo-Christensen, and Marten Landahl, and it served as an engineering training source for many of the industry's aeroelasticians. A more complete story of the laboratory and of the individuals involved is given by S. Ober.[25]

Air Racers Encounter Flutter

After World War I, highly competitive attempts to break world speed records and to capture coveted air-race prizes stimulated designers to push for ever higher speeds. A price paid for this otherwise healthy rivalry was a series of flutter encounters, usually catastrophic, which occurred during high-speed runs. Shown in Fig. 8 are some of the racers which are known, or believed to have been, plagued by flutter problems. In the 1922 Pulitzer Trophy Races the wings of two cantilever monoplane entries, the Loening P-4 and the Verville Sperry R-3, had to be hastily stiffened in order to resist this heretofore little known but highly dangerous phenomenon. The flutter cure included, among other things, covering the wings back to the rear spar with stiff plywood veneer. Later, other builders of wooden cantilever monoplanes adopted veneer wing covering as a means of providing the torsional rigidity needed to avoid flutter; this prompted Bill Stout, builder of the all-metal Ford Trimotor which was flutter-free to quip,... flutter is a "Veneer-eal disease."

During the 1924 Pulitzer Race in Dayton, Ohio, the Army entry, a Curtis R-6 racer, developed sudden vibrations, then shed its wings in a steep dive at the very start of the race. There is some uncertainty here whether the breakup was caused by wing flutter or by failure of the laminated wooden propeller. The following year, Great Britain's prized entry in the 1925 International Schneider Marine Trophy Race, a Supermarine S-4 Racing Monoplane, experienced wing flutter during a prerace trial flight and crashed into the Chesapeake Bay at Baltimore. The pilot, who just managed to survive, said the wings "fluttered like a moth's wings." It is significant to note that the S-4 was an unbraced cantilever-wing design; after its crash the designer, R.J. Mitchell, who later designed the Spitfire, reverted to externally braced wings for the Supermarine S-5, S-6, and S-6b, which went on to win three Schneider trophies and two World Speed Records during the 1927-1931 period. Later, in an attempt at breaking the world's landplane speed record in 1931, a Gee Bee racer and its pilot were lost in what has been attributed by some to wing-aileron flutter during the high-speed diving start.

In some incidents, where flutter was more forgiving, the pilot and his plane would return "shaken" but unharmed. When given this second chance to correct his design, the builder would sometimes resort to bold, if not imaginative solutions. Leon Tolve describes one such incident in a historical account of flight flutter testing[26]: "In 1934, during the National Air Races, one of the racers kept encountering wing-tip flutter. Each time the wing span was reduced by cutting off part of the wing tip until the flutter stopped. As a result, the wing area was finally reduced from its original value of 78 ft^2 down to 42 ft^2, but the pilot ended up with a flutter-free airplane!"

1930 to World War II

British Studies

In contrast to its foresight in 1924-1925, the predictions of the British ARC in the 1929-1930 yearbook completely missed the mark with its statement, "The Committee considers that the main practical issues of the subject of wing flutter have now been put on a satisfactory basis and that, from a purely practical standpoint, there does not seem any need to pursue the theory further." In fact, the 1930s were a decade of considerable ferment and progress in aeroelasticity, especially in the theory. The structurally stiff biplane had lost ground to the monoplane with its superior performance; the fabric covered wings of wood spar construction were being replaced by metal covered wings of semimonocoque construction with metal spars and internal stiffeners, and of monocoque construction wherein the skin covering provided a large part of the stiffness. Moreover, speeds were approaching an appreciable fraction of the speed of sound (see Hoff, Survey Papers, 1967).

In 1932, a series of accidents with fatalities was encountered by the de Havilland Puss Moth airplane, a general purpose single-engined monoplane, whose wings were braced by folding V-struts. A 1936 comprehensive report of the ARC Accidents Investigation Subcommittee[27] summarized more than 50 separate detailed investigations. Of special interest was the conclusion that not only wing flutter but also rudder and elevator flutter may have been involved, that the V-struts were a factor in the wing flutter, and that the rudder flutter seemed to require a starting impulse such as stormy or turbulent weather.

Table 1 Some U.S. flutter experiences, 1932-1934

Airplane	Type of flutter
YC-14 (General Aviation)Transport	Wing-aileron[a]
C-26A (Douglas) Transport	Wing-aileron
XO-43 (Douglas) Observation	Wing-aileron
YO-27 (General Aviation) Observation	Wing-aileron[a] (free play a factor)
YO-27 (General Aviation) Observation	Rudder-fuselage[a] (violent flutter)
F-24 (Fairchild) Civil	Wing-aileron
F-24 (Fairchild) Civil	Tail flutter
YA-8 (Curtiss) Attack	Rudder fin[a]
YB-9A (Boeing) Bomber	Rudder-fuselage (limited amplitude flutter)
XV-7 (Douglas) Bomber	Elevator-fuselage[a] (elevator interconnect-stiffened)
YO-40B (Curtiss) Observation	Elevator-tab[b]

[a]Solution: mass balance. [b]Solution: increased tab frequency.

Duncan and Collar[28] extended the theory of Glauert to include wing translation and rotation, and, like Glauert, they obtained their results by numerical integration. Cox and Pugsley[29] and Duncan and MacMillan[30] investigated the newly discovered aeroelastic control-problem aileron "reversal" wherein, as the speed is increased, the deflection of the ailerons produces wing twist opposing the ailerons, so that the rolling power or effectiveness of the ailerons diminishes, may vanish, and then act in the opposite direction. Although it is not an instability problem per se, the control problem can be dangerous.

The British were not alone with flutter problems. During 1932-1934, there were many flutter cases in the U.S. Table 1 lists some of these. The information was supplied to the authors by Leon Tolve.

Theodorsen: Two-Dimensional Flutter Theory

In the U.S., Theodore Theodorsen attacked the flutter problem in 1934 and within a few months of intensive concentration produced NACA Rept. No. 496,[31] which has played a large role in establishing methods of flutter analysis in American aircraft industry. (Garrick worked closely with Theodorsen over the period 1930-1946, and has described Theodorsen's many contributions to flutter and other areas in a separate article.[32]) Theodorsen gave a succinct theory of the two-dimensional oscillating flat plate undergoing translation, torsion, and aileron-type motions. He separated the noncirculatory part of the velocity potential from the circulatory part associated with the effect of the wake. Again the trailing-edge flow condition sets a relation between the two parts, whose solution leads to a combination of Bessel (or Hankel) functions designated $C(k)$. This function establishes the lags between the airfoil motions and the forces and moments that arise, and has been denoted as Theodorsen's (circulation) function $C(k)$, where, analogous to the Strouhal number, k is a reduced frequency, $\omega b/V$ (ω is the angular frequency, b the half-chord, and V the airspeed). The quasistationary constants used in earlier work thus become frequency dependent. Because the various phases and lags are crucial in determining whether energy can be extracted from the airstream, that is, whether flutter can occur, Theodorsen's theory represented the simplest exact theory for the idealized flat plate airfoil, and has served a major role in so-called "strip" theory wherein representative sections are employed in wing flutter analysis.

Theodorsen's method of solution for the flutter stability equation differs from that of his predecessors, in that he makes no use of Routh's discriminants; for, as he deals with sinusoidal aerodynamics, the determinant whose vanishing yields the eigenvalues is complex so that both its real and imaginary parts must vanish simultaneously to determine a flutter condition. This leads to several parametric ways of finding the flutter solutions. Both binary and ternary types of flutter were studied.

Theodorsen's work has played a major role in the development of American methods of flutter analysis, in the relatively simple use of strip theory for wings of moderately high aspect ratios, and in several other approximate procedures. It must be stated that although Theodorsen made no direct use of the work of Wagner and Glauert and gave no references, his method of analysis is clearly related to their work. This circumstance may have left resentments, and may have a bearing on the divergence of U.S. and British methods of flutter analysis.

Theodorsen and Garrick[33] developed numerous applications and trend studies of the simple exact theory yielding insights into the individual effects of the many parameters: center of mass, elastic axis, moments of inertia, mass ratio (mass of the wing to a surrounding cylinder of air), aileron hinge location, bending/torsion frequency ratio, etc. In particular, the material damping, represented by hysteresis damping (g), which is obtained by multiplying the elastic restoring force by the factor $e^{ig} \approx 1 + ig$, was also varied. Experimental studies on simple cantilever models of high aspect ratio confirmed the basic theory with good agreement. Aileron flutter, which often can occur only over a limited speed range, was also confirmed. It was shown that for a wing of high aspect ratio the flutter mode could involve much second and higher bending modes. The confirmation of the Theodorsen theory by means of flutter speed measurements is an indirect process. A direct experimental confirmation of the oscillating lift of an airfoil in pitching motion was made by Silverstein and Joyner.[34]

It is appropriate to record that, starting about 1935, flutter was a topic of discussion appearing in Japanese, Russian, French, and Italian papers, as indicated in the bibliography in Ref. 35. Illustrating the classic pattern of the evolution of ideas when the time is ripe, Placido Cicala[36] in Italy, following Birnbaum's method, obtained an independent solution of the oscillating flat plate only months after Theodorsen, while Küssner[37] developed a solution in the following year (1936) in a paper which also summarized the state-of-the-art in the development of the theory. In his paper Küssner gave the method for obtaining the gust function denoted by $k_2(s)$, the growth of lift following entrance of the flat plate airfoil into a sharp-edged gust. An error in sign in the derivation was corrected by von Kármán and Sears.

Propulsion of Flapping Wings and Aerodynamic Energy

It is a source of satisfaction to aeroelasticians that their field has contributed to the understanding of the age-old problems of the flight of birds and locomotion of fish. The first hint of this is in an application of Birnbaum's theory to the calculation of the horizontal forces on a flapping wing by J. M. Burgers,[20] who developed the theory of the propulsive forces on a flat plate airfoil including the effect of the suction force at the sharp leading edge, which, paradoxically, holds for the rounded edge. Similarly, Garrick treated the flapping and oscillating airfoil with aileron.[38] During more recent years T.Y. Wu has developed at the California Institute of Technology a more complete theory, while Lighthill has brought in biological studies and introduced a new field termed "biofluid-dynamics." The flow of blood in elastic arteries is another example of this field, as is the "hydroelasticity" of planing surfaces. The aerodynamic energy required to maintain the motion of the oscillating wing in the airstream has also been a useful concept in flutter analysis. Perhaps the simplest physical picture of the mechanism of flutter is arrived at through the aerodynamic energy, as by Duncan's "flutter engine" wherein one could extract energy from the oscillating airfoil in the wind tunnel by means of a crank and flywheel (see Survey Papers, Duncan, 1951 and Nissim, 1971).

Oscillatory/Indicial Aerodynamics

There are interesting and important relationships between oscillatory and indicial aerodynamics, noted by Garrick,[39] that are analogous to those of electric circuit analysis between the frequency response function to alternating current and the Heaviside response to unit step excitation. These relations correspond in the simple flat plate cases to Fourier integral or Laplace transform relations between Wagner's function $k_l(s)$ and Theodorsen's function $C(k)$. Similar relations can be developed for more general configurations; they rest essentially on the validity of the principle of superposition for linear processes. Such spectral techniques are useful in other areas involving transfer functions, for example, in the determination of the response to gusts and turbulence.

In addition to the correction of the Küssner gust function $k_2(s)$ mentioned above, von Kármán and Sears[40] at the California Institute of Technology developed another independent treatment of the oscillating flat plate in incompressible flow. The transfer function of $k_2(s)$, given by a combination of Bessel functions, has been designated Sears' gust function and is made use of by him in several gust studies.[41] These reciprocal relationships between the indicial lift functions of Wagner and Küssner and the counterpart "frequency response" functions of Theodorsen and of Sears are graphically illustrated in Fig. 9. A. E. Lombard, another doctoral student of von Kármán, gave in his thesis (1939) many numerical applications and a summary of the literature. The esoteric and abstract mathematical nature of flutter analysis gave the subject an atmosphere of mystery, magic, and skepticism in the design office, and led von Kármán to remark (as quoted by W. Liepmann), "Some fear flutter because they do not understand it, and some fear it because they do." Indeed some designers would not become true believers until confronted by flutter occurring in their own designs.

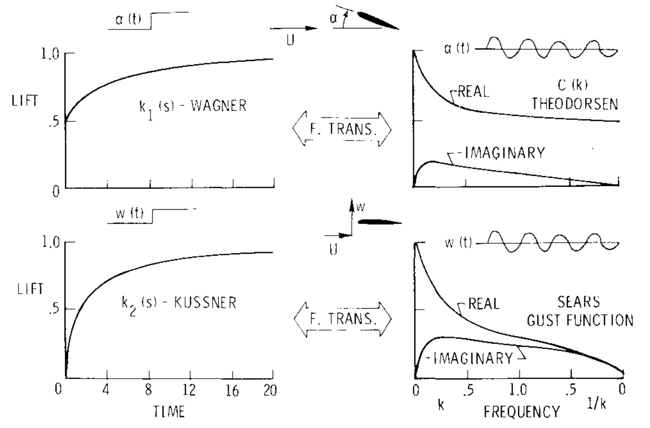

Fig. 9 Fourier transform relationships between oscillatory and indicial aerodynamics. (Modified Sears function is referred to leading edge rather than to midchord. See Giesing et al. *Journal of Aircraft*, 1970.)

Aerodynamic Hysteresis

The theory of Wagner on the growth of circulation or lift following a change in angle of attack was well demonstrated in a water tank by P.B. Walker[42] working with W.S. Farren. Farren examined experimentally the increase in lift coefficient for a wing whose angle of incidence is changing rapidly, and showed that the lift coefficient could increase well beyond maximum lift.[43] This phenomenon leads to complex nonlinear hysteresis effects for an oscillating airfoil, which are functions of the Reynolds number (Fig. 10). The phenomenon occurs in stall flutter of wings, propellers, and rotors, and in high angle of attack buffeting.

Experimental work at low speeds on the effects of angle of attack on aeroelastic phenomena was started in 1936. The effect of high angles on the flutter speed, an inherently nonlinear problem, was begun by J. Studer[44] under direction of Professor Ackeret. A similar study of lesser scope was undertaken by Rauscher[45] about the same time at MIT. The significant result was obtained that with increase in angle of attack the coupled wing flutter speed dropped markedly in the neighborhood of the stall, and became essentially single-degree-of-freedom torsional flutter, a less violent type. The prevention of this phenomenon is currently important for all types of high lift devices, for rotor wings, and for turbomachinery; it sometimes may confound the buffeting picture for high angles of attack associated with wake excitation and vortex separation.

Empirical Criteria

In 1935, Küssner correlated many flutter incidents and accidents, and developed an empirical formula based on the reduced torsional frequency ($\omega b/V$), a criterion which gave only a ball park estimate of the flutter speed for the then current types of aircraft. A similar statistical study had been made by Roxbee Cox (1933), however, based on wing torsional stiffness instead of frequency. It is interesting that these two empirical formulas, one based on natural frequency and the other on stiffness, actually reflect the separate paths along which flutter theory had evolved. Whereas the German approach, typified by Küssner and also by Theodorsen, considered the flutter motion as sinusoidal expressed in terms of natural frequencies, the British approach proceeded along the lines of Bryan's stability theory using Routh's discriminants and involved the stiffness, inertia, and damping coefficients that appear in the equations of motion. As a consequence, British researchers in the early stages of flutter development tended to overlook any direct connection between natural vibration modes and flutter. During this period, resonance testing in England was neglected in favor of stiffness measurements which were considered to be more directly applicable to needs. By 1936, however, resonance testing in connection with flutter investigations of aircraft and wind tunnel models had become as accepted in England as elsewhere.

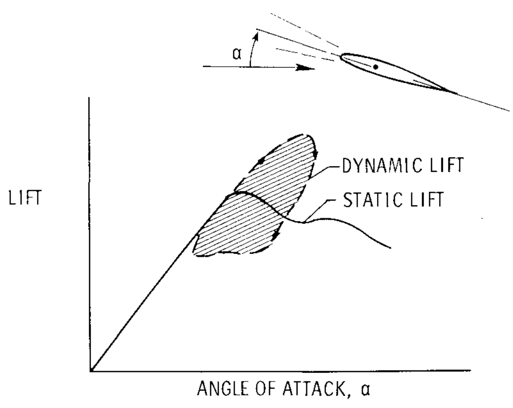

Fig. 10 Nonlinear behavior and hysteresis loop near stall for oscillating airfoil.

Flight Flutter Testing

In 1935 in Germany, von Schlippe became the first to employ resonance testing techniques in flight.[46] The purpose of his research was to lessen the extraordinary risks involved in testing in flight for flutter safety. The usual procedure for flight checking a new or modified design had been simply to dive the airplane to its maximum velocity and hope for the best. In justifying his work, von Schlippe stated the problem in much the same way we would describe it today: "Lately the problem of spontaneous oscillations has become particularly acute as a result of the trends in modern airplane design. Thinner profiles, divided tail units, greater number of cut away sections in the wings, as well as the generally great weight of the fast airplanes, are all factors which, with the demands for greater speeds, reduce the range between critical and maximum velocity, and through it promote the danger of oscillation."

The basis of the von Schlippe method is that at the critical flutter speed the resonant amplitude response of the airplane structure to forced oscillations would be infinitely large, unless modified by nonlinear effects, so that a plot of the resonant amplitude against airspeed would have an asymptote at the flutter speed (Fig. 11). The estimated position of the asymptote, and hence of the flutter speed, could then be deduced from observations of the forced amplitudes at airspeeds below the critical speed. However, from wind tunnel and theoretical studies of von Schlippe's method, Frazer and Jones[47] cautioned that under certain conditions the damping could drop very suddenly near the flutter speed for very small changes in airspeed. The Germans successfully carried out systematic flight flutter tests using the technique on a series of aircraft, until in 1938 a Junkers JU90, equipped with a 400-hp motor in the fuselage to drive vibrators in the wings, fluttered unexpectedly during the flight tests and crashed.

Because of the hazards of flight flutter testing, there was a strong reluctance by aircraft manufacturers to perform such tests. Nevertheless, it was recognized by some that if it was dangerous to conduct a flight flutter test, then it would be far more dangerous to fly without it. By the late 1940s flight flutter testing began to gain acceptance by the industry as a result of improvements in testing techniques and flight instrumentation, along with a better theoretical understanding of the flutter problem. Methods for flight flutter testing have evolved into very advanced procedures utilizing flight and ground based digital computers, real time test and analysis, and a variety of methods of excitation, steady-state, transient, pulsed, and random (see Survey Papers, NASA SP-385 and SP-415, 1975).

Propeller Whirl Flutter

In 1938, in a study of vibration isolation of aircraft engines, Taylor and Browne[48] examined the possibility of a new form of instability that later came to be known as propeller whirl flutter. Unlike propeller-blade flutter, a cousin of wing flutter, propeller whirl flutter involves the gyroscopic precession of a flexibly mounted engine-propeller system. Because the conditions necessary for such an instability were never encountered in aircraft designs of that time, the problem was considered to be of academic interest only. However, after remaining dormant for more than two decades, propeller whirl flutter was suddenly "rediscovered" as being the probable cause of the crashes of two Lockheed Electra turboprop transports (see Survey Paper, Reed, 1967). The instability was attributed to a severe reduction in nacelle support stiffness due to some form of damage in the engine mount structure. In undamaged condition the aircraft had an ample margin of safety from whirl flutter. The cure involved among other things stiffening and redesigning the mount system to make it "fail-safe." Whirl flutter stability has also become an important design consideration for prop-rotor VSTOL aircraft.

Matrix Methods

One of the timely publications of the prewar period (1938) was a unique textbook by Frazer, Duncan, and Collar[49] on matrices and their applications, spiced with several flutter examples. By this time the simple binary and ternary cases were needing expansion to include many additional structural degrees of freedom. In 1941, S. J. Loring[50] gave a prize paper outlining a general approach to the flutter problem that made convenient and systematic use of matrices. The expansion of numerical effort soon to become overwhelming by earlier methods required the employment of systematic procedures afforded by matrix methods, both in structures and in aerodynamics. Matrix methods also fitted in with the parallel growth in the use and capacity of computing machines that was to evolve during the next decade. Figure 12 sketches a matrix form of the flutter equations.

Compressibility Effects

About the mid-1930s the effects of airplane speeds increasing to near sound speed were becoming important; that is, flight Mach numbers were such that local Mach numbers approached one. A significant paper by Prandtl[51] in 1936 on steady aerodynamics in a compressible medium set the stage for its rapid generalization to unsteady aerodynamics. Prandtl introduced the useful concept of the acceleration potential, in contrast to the usual velocity potential. The concept has been useful however mainly for the small-disturbance linear-theory approach. In this case the acceleration potential is a special grouping of velocity potential terms $(\partial\phi/\partial t)+V(\partial\phi/\partial x)$ representing the normal pressure. Since there is no pressure loading across the wake, this term vanishes there, and the wake boundary condition is accordingly simplified. Prandtl's theory holds well for both

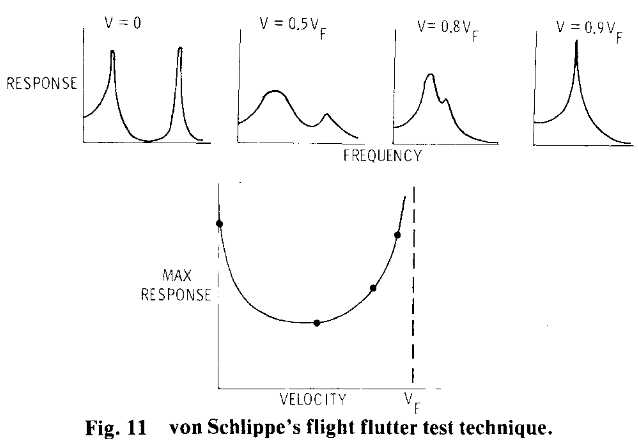

Fig. 11 von Schlippe's flight flutter test technique.

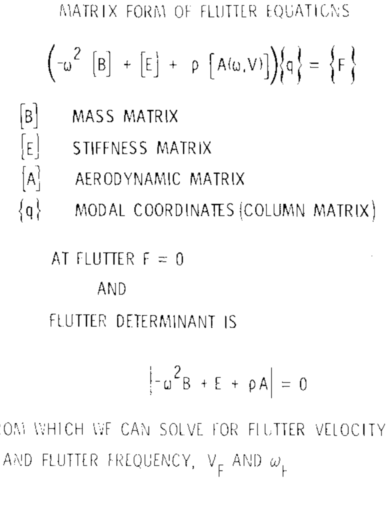

Fig. 12 Matrix methods enable systematic calculations of aeroelastic effects.

small disturbance subsonic ($M<1$) and supersonic ($M>1$) speeds, however, the linear theory is not valid for the transonic and hypersonic speed ranges.

There followed shortly afterward two short outstanding contributions by Camille Possio in Italy. In 1938,[52] he applied the acceleration potential to the two-dimensional nonstationary problem and arrived at an integral equation (Possio's equation), the solution of which gives the loading over a flat plate airfoil in the airstream for a known motion of the plate, i.e., for a given downwash. Possio indicated a procedure for its numerical solution, although several others later contributed more convenient methods. Possio also gave an outline of the parallel problem for a supersonic mainstream.[53] Possio's brief brilliant career ended with his death during the war years.

Finite Span Considerations

In the case of steady flow about finite wings, Prandtl's integral equation for determining the induced drag and the spanload distribution played a well-known and important role. It was natural to try to generalize this approach for unsteady flows while keeping the two-dimensional exact theory as a limit for the infinite aspect-ratio wing. This resulted in what had been referred to as lifting-line and multiple lifting-line methods for oscillating wings of finite span. The first of these methods was developed by Cicala[54] (1937). Other related work of interest was given by W. P. Jones[55] (1940), Küssner[56] (1943), and Reissner[57] (1944). These methods played an important interim role until computational methods applicable to true lifting surface methods evolved some years later. R. T. Jones[58] contributed to this area from the viewpoint of the indicial aerodynamics. He gave an approximation for the Wagner function in a form useful in applications with transfer functions:

$$k_1(s) = 1 - a_1 e^{b_1 s} - a_2 e^{b_2 s} \ldots$$

and gave similar developments for finite elliptic wings of various aspect ratios.

General Lifting Surface Theory

The basis for a general lifting surface theory for finite wings was given by Küssner[59] in a classic paper published during the war, issuing from his newly formed Institute for Nonstationary Phenomena in Göttingen. (The unique issue of the journal in which this paper appeared was devoted to nonstationary aerodynamics, and included other papers of lasting interest.) Küssner made direct use of Prandtl's acceleration potential and of the effect of a uniform moving doublet to obtain an integral equation of the form

$$w(x,y) = \int\int L(\xi,\eta) K(x-\xi,\ y-\eta)\, d\xi d\eta$$

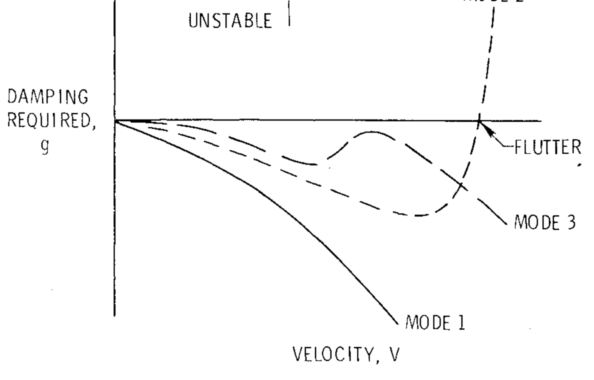

Fig. 13 Flutter solution by the *V,g* method.

The equation relates the unknown load distribution L over the lifting surface and the known velocity normal to the surface, the downwash w by means of a quantity K, known as Küssner's kernel function, which represents the normal downwash induced at any point by a unit point load. The function K depends on the retarded solution of the acoustic wave equation and holds for the subsonic range. It was left by Küssner in the form of a highly singular integral whose solution could be found in special cases. For example, in two dimensions it reduced to the kernel of Possio's equation. It was not until 1954 that a general explicit expression for K was developed at the NACA Langley Aeronautical Laboratory by C. E. Watkins, H. L. Runyan, and D. S. Wollston[60] which opened the way to fuller development of appropriate methods of solution of the integral equation, as the primary basis and focus for numerical methods for the aerodynamics needed in flutter analysis.

It is pertinent to remark that numerical lifting surface methods utilizing the velocity potential are also in contention; for incompressible steady flow, discrete numerical methods were initiated by V. M. Falkner[61] and were extended to oscillating flows by W. P. Jones.[62] Very recent work of Morino (see Survey Paper, 1974) hinges on such methods, which can go beyond the linear approximation.

World War II to the Mid-1950s

During World War II, rapid changes took place in airplane development. The trend toward higher speeds and toward all-metal aircraft persisted. Fighter aircraft and long-range bombers of diverse configurations, of low and high aspect ratios, carried external armament, tip tanks, and other appendages. A tip-tank flutter problem, for example, occurred on the P-80 airplane. Flutter problems occurred in the field due to appendages or battle damage which could cause loss of balance weights or reduced stiffness.

The *V,g* Flutter Diagram

As an aid to flutter analysis and practice, Smilg and Wasserman[63] in a 1942 document gave comprehensive tables of unsteady aerodynamic coefficients based on the theory of Theodorsen, and supplemented by tables on control-surface aerodynamic balance from Küssner and Schwarz (1940). These tables, together with the suggested computational procedures, served as the handbook for flutter analysis in the United States for a number of years.

Their flutter computation procedures adapted the structural damping concept involving the parameter g to give a useful way of graphically exhibiting a flutter solution by means of the V,g flutter diagram. In this commonly used procedure, with abscissa the speed V, and ordinate the damping factor g, the flutter solution is conveniently represented by the crossing of the $g=0$ axis by a particular frequency branch (Fig. 13). In this sketch mode 2 shows a flutter crossing, while mode 3 indicates a low damping sensitivity. Each point in this representation is a harmonic flutter solution with an artificial damping g, namely, that required to sustain harmonic motion. The appropriate flutter solution is that which corresponds to the actual damping in the structure, often taken as $g=0$. The actual damping in each mode can be modeled and included by means of separate g's.

Unsteady Aerodynamic Measurements and Aeroelastic Models

While significant strides had been made in the advancement of aeroelastic analysis, researchers and designers continued to place heavy reliance on obtaining complementary experimental data. Aeroelastic model tests in wind tunnels, supported by mathematical analysis, gave designers that much needed feeling of confidence that neither theory nor experiment alone could provide. These wind-tunnel investigations ranged from measurements of the oscillating airloads to flutter-proof tests using complete aeroelastic models of prototype aircraft. In addition to providing designers with solutions that might not be obtainable by theory in a reasonable length of time, such experiments also are extremely useful tools for evaluating and guiding the development of theory.

In a survey of oscillating aerodynamic derivative measurements during the years 1940-1956, Hall[64] cites 53 published studies, conducted mainly by British and U.S. investigators. This survey revealed that much of the work during the war years was done by the British, however, at low subsonic speeds; after the war, in the U.S., the main emphasis was on obtaining results for the transonic and supersonic speed ranges.

Since the earliest attempts in the twenties at measuring air loads on oscillating surfaces it was realized that the development of reliable testing techniques was important and difficult to achieve. Early methods for measuring amplitude and phase of the pitching moment of an oscillating airfoil relative to its motion were often laborious and inaccurate, involving the reading of photographic records of vibration time histories contaminated by extraneous noise and vibration. An ingenious yet simple electrical measurement technique, which overcame these difficulties, was developed by Bratt, Wight, and Tilley.[65] Known as the "wattmeter" harmonic analyzer, the technique made it feasible to greatly expand the scope, and improve the accuracy of oscillatory aerodynamic derivative measurements.

The wattmeter analyzer in combination with the Kennedy-Pancu vector method of vibration measurement and analysis[66] played an important role in other areas of dynamic testing, such as ground and flight resonance testing of aircraft; it may be regarded as a forerunner of present-day vibration analyzer instruments.

The usefulness of wind-tunnel flutter-model tests to validate theory, study flutter trends, and determine margins of safety for full-scale prototypes had already been well established for low-speed aircraft of the early thirties. A decade later, with flight speeds approaching that of sound, and aircraft of all-metal construction, new requirements arose for the design and fabrication of aeroelastically scaled flutter models. During the war years a popular method of constructing low-speed flutter models for use in the proof testing of prototype designs utilized a readily workable plastic, polyvinyl-chloride. This material, having a density and elastic modulus much less than that of the full-scale aluminum aircraft structure, permitted the internal as well as the external construction of the model to be geometrically similar to that of the prototype structure; that is, it made replica-type construction feasible. Experiences with such models are described by Wasserman and Mykytow[67] (see Survey Papers, Williams, 1951). We may note that as early as 1938 a ¼-scale complete replica wing model of the PBM-1 seaplane was tested in the NACA Propeller Research Tunnel. Investigators were Felix Nagel, William Bergen, Rene H. Miller, and Edwin P. Hartman. The tests in this instance disclosed a wing divergence speed very near to the wing flutter speed. A flutter model of this kind, tested about 1942, was of an unconventional fighter design, the Vultee XP-54. With a pusher propeller on the aft fuselage and twin booms extending aft from the wing to support the horizontal and vertical tails (Fig. 14), it was expected that this configuration might have some unusual flutter problems. Elevator flutter was, in fact, detected during wind-tunnel tests and the problem corrected in time to eliminate its occurrence on the airplane. In Germany, a complete aeroelastically scaled replica-type plastic model of the Junkers JU-288 was tested in 1944. The model, with a wing span of more than 7 m, was flexibly suspended by wires in the wind tunnel to enable simulation of rigid-body free flight modes. Results from these tests were employed to guide selection of such design variables as the stiffness and mass balance of control surfaces, empennage connection stiffness, and also engine mounting stiffness (Fig. 14). These tests probably represent the first flutter-model experiments in which simulation of free flight was attempted (see Survey Paper, Biot, 1945).

The popularity of replica-type plastic flutter model construction eventually waned because of limitations in manufacturing tolerances, changes in material properties with temperature and humidity, and high cost of fabrication. The replica model concept was replaced by a much simpler model-design approach wherein only those modes of vibration expected to be significant from the standpoint of flutter were represented by the model. With this approach, beam-like wings could be simulated by a single metal spar having the proper stiffness distribution to which were attached aerodynamic contours in the form of light balsa wood pods. The correct mass and moment of inertia at each spanwise station could be matched by means of weights installed in the pods. Model studies of this kind, as shown in Fig. 14 for the B-47, were instrumental in guiding the flutter design of the B-52 and the jet transports that evolved from it.

For research purposes models can be far less complex than the elaborate development-type models described. Indeed, the simplest model that enables study of the particular phenomenon of interest is usually the best model. A compilation of experimental flutter research in the U.S. using simplified wing and wing-aileron flutter models, covering the postwar period through 1953, is documented by Cunningham and Brown.[68] Other examples of the role of wind-tunnel models in aeroelastic research prior to the mid-fifties are given by Templeton[69] for low-speed models, by Targoff and White[70] for transonic models, and by McCarty and Halfman[71] for supersonic models.

A special 4-ft wind tunnel designed exclusively for flutter research at high subsonic Mach numbers (up to about 0.8) became operational at the Langley laboratory in 1946. A novel feature of this tunnel was in the test medium, which could be either air or freon gas and which could be varied over a ratio of 30 to 1 in density. The freon test medium is particularly desirable for scaled flutter-model testing at subsonic and transonic Mach numbers because of its higher density by a factor of 4, and lower speed of sound by about one-half, compared with that of air. This tunnel, later modified by means of a slotted throat to give transonic capability, became a precursor for the Langley Transonic Dynamics Tunnel, which today is the key facility in the United States dedicated to experimental investigations in the field of aeroelasticity.

Transonic Flutter Problems

With the advent of flight at transonic speeds brought about mainly by the jet engine, came a host of new and challenging aeroelastic problems, many of which remain to this day, as the transonic speed range is nearly always the most critical one from the standpoint of flutter. One such problem to capture the attention of aeroelasticians was a violent form of aileron oscillations encountered in 1944 by NACA pilots during high-

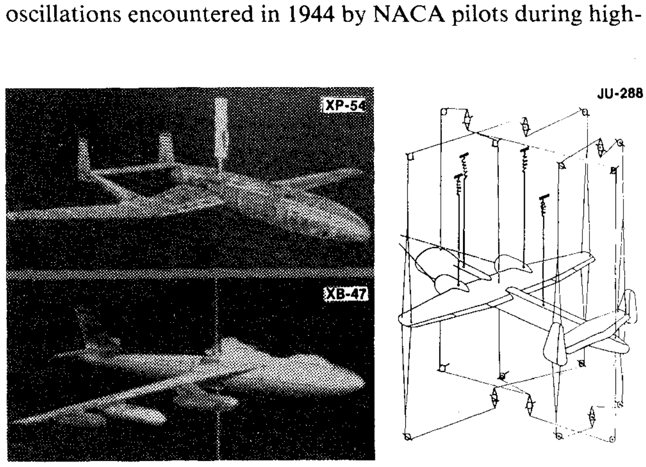

Fig. 14 Flutter models of prototype aircraft in wind tunnels.

speed flight tests of the new P-80 airplane. This phenomenon, called "aileron buzz," was identified as a single-degree-of-freedom type of flutter caused by the coupling of aileron rotation and chordwise motion of shock waves on the wing. As first described qualitatively by Erickson and Stephenson,[72] it is attributed to the aerodynamic lag effects over the control surface associated with the shock location and its movement as affected by speed and angle of attack (see also Smilg[73]). The phenomenon may also involve periodically separated and reattached flow behind the shock. First attempts at eliminating the problem were along lines known from past experience to be effective for other forms of aileron flutter, for example, by the use of control-surface mass balance. Wind-tunnel tests in the Ames 16-ft tunnel showed that mass balance, even large amounts of overbalance, had virtually no effect on the severity of the oscillations. Eventually, after extensive flight and wind-tunnel investigations (as there were no suitable transonic theories), practical means of dealing with the problem evolved. The solutions included increased control stiffness, dampers, and profile shape changes.

An empirical criterion for the avoidance of transonic control-surface flutter was given by Arthur A. Regier (Survey Paper, *AGARD Manual,* Vol. V) in the simple form $(\omega_\beta c_\beta / 2a) > 0.2$ to 0.3, where ω_β is the control-surface frequency, c_β the control-surface chord, and a the sound speed. A similar criterion of Regier for avoidance of propeller stall flutter was $(\omega_\alpha c/2a) > 0.4$, where ω_α is the blade torsional frequency and c the chord.

Following concepts first given in 1945 by R.T. Jones (and for supersonic flow, earlier by A. Busemann), sweep as a design feature to allow efficient penetration of transonic speeds was introduced. The Boeing B-47 bomber made use of sweep and aeroelastic tailoring to produce a highly efficient design for the time. The placement of the nacelles on the wings was dictated by conditions for avoidance of flutter, as well as for reduced wing-root bending moments. Since the wings were highly flexible (there was a demonstrated 30-ft difference in tip deflection for maximum up and down loads), careful design was needed to obtain the proper jig shape to achieve the desired flight characteristics. Because sweep was to become almost universally used, the effects of sweep on flutter opened a new dimension and became an important consideration from both structural and aerodynamic points of view.

The need for thin wings for high-speed aircraft compounded the difficulties in meeting stiffness requirements to avoid transonic flutter; moreover, the nonlinear flow theory was lacking, and transonic wind tunnels did not yet exist. It became imperative therefore to develop alternative suitable experimental means for investigating flutter in this critical speed range. Four methods for meeting these needs by means of aeroelastic models evolved (Fig. 15). Two of the methods made use of free flying models. With wartime developments of radar and telemetry at hand the Langley Laboratory, in 1946, began experimental flutter studies in the transonic range by means of models dropped from high flying aircraft and by ground-launched rocket-propelled models (see Survey Papers, Shortal, 1978). In another method, the wing-flow technique, the model was placed on the upper surface of an airplane wing in a region of nearly uniform transonic flow. Also, rocket sleds capable of accelerating to transonic speeds were used in flutter investigations both for models and full-scale components.

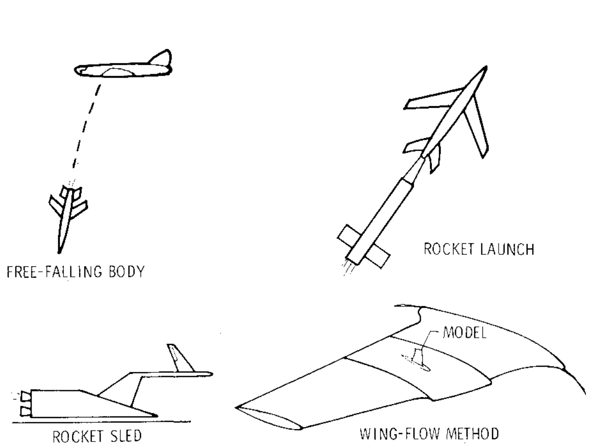

Fig. 15 Early transonic flutter experimental methods.

By the early fifties transonic wind tunnels had become a reality, because of development at Langley of vented test-section walls for which much credit is due John Stack. Flutter experiments at transonic Mach numbers could now be performed in wind tunnels, and with much greater efficiency and less cost than by the methods mentioned. At Langley the 4-ft Freon Tunnel was converted to a 2-ft continuous flow transonic tunnel and used in flutter research. Also, a 26-in. transonic blowdown tunnel became operational and, because of its versatility and economy, was particularly useful in aircraft development work employing small wing and tail flutter models.

Systematic flutter tests at supersonic speeds were begun by Regier about 1950 with the use of a small blowdown tunnel. To avoid the initial shock characteristics of such tunnels, he developed a technique for injecting the model after the initial shock had passed, and uniform flow was established. Later a similar technique, involving initial restraints, was used in the 9- by 6-ft ($M=3$) Thermal Structures Tunnel which was used for many flutter tests. These included actual components of the X-15, an airplane which flew above $M=7$ and reached 300,000-ft altitude. During development it had several panel flutter problems.

Flutter at Supersonic Speeds

Flutter at supersonic speeds began to be studied more seriously as speeds in dives could readily become supersonic. Supersonic speed in level flight was achieved in 1947 by Charles Yeager in the X-1 research airplane. Analytical flutter studies were undertaken a few years earlier by Collar and by Temple and Jahn[74] in England, by von Borbely[75] in Germany, and by Garrick and Rubinow[76] in the U.S., expanding on Possio's work. It is of interest that the potential for supersonic speeds is composed equally of both the retarded and advanced potential forms as noted by Küssner (see Survey Papers, 1950) and by Garrick (see Survey Papers, Princeton University Press, 1957). Although, in general, because of the rearward shift of the aerodynamic center, classical coupled flutter seemed less likely to occur, because of the changes in altitudes of flight and in configurations, especially sweep, flutter could not be ruled out. Moreover, the nonlinear effects of thickness are more pronounced at supersonic than at subsonic speeds, as indicated by the simple piston theory approach of M. J. Lighthill[77] wherein the pressure and local velocity become point functions of each other. The single-degree-of-freedom negative damping in a pitching motion uncovered by Glauert in 1929 for certain cases in two-dimensional flow persists into the subsonic and lower supersonic ranges. Fortunately, it is alleviated by damping and by span effects.

A new type of flutter, panel flutter, could occur involving the skin covering wherein standing or traveling ripples in the skin persisted, which could often lead to an abrupt fatigue failure. Panels are natural structural elements of both aircraft and spacecraft so that avoidance of panel flutter is important. Panel flutter depends on many parameters, including the Mach number and the boundary layer, but especially on any compressive or thermal effects that tend to create local buckles in the skin. Wernher von Braun informed one of the authors that more than 70 failures of the V-2 rocket occurred during the war in the period when it underwent development and test. Many of these failures were found to be caused by

Table 2 Flutter incidences for U.S. military aircraft, 1947-1956

Type of flutter	No.
Tabs	11
Control surfaces	26
Wings	7
Tails	7
Other	3
Total	54

flutter of a panel near the nose of the rocket. In the 1950s a fighter airplane was lost in a test flight because of failure of a hydraulic line which had been attached to a panel that fluttered. In another group of fighter airplanes a serious cockpit noise problem was solved after being traced to panel flutter. In the 1960s, the panel flutter problem for the Saturn V Apollo launch vehicle required costly investigation (for references see Survey Papers, Goodman and Rattaya, 1966 and the *AGARD Manual*).

Flutter Incidences

The seriousness of flutter during the ten-year period following the war is indicated by a survey of flutter encounters compiled by the NACA Subcommittee on Vibration and Flutter in 1956 in a state-of-the-art survey[78] (Table 2). This listing indicates that a total of 54 flutter difficulties have been documented for various components on U.S. military aircraft. Although the table is far from being complete and does not include commercial or civilian aircraft, it clearly indicates the types of flutter problem areas facing designers of that period. For example, it is significant that 21 involved transonic control surface buzz, for which no reliable theory or basic understanding was available for guiding design. All seven of the wing flutter cases were associated with externally mounted stores including pylon-mounted engines, a problem area of much concern even today. The early suspicion that all-movable control surfaces (needed for adequate control at transonic and supersonic speeds) would be a source of new flutter difficulties was confirmed by the occurrence of four such flutter encounters in two years during the latter part of the survey period.

During the early fifties general aviation aircraft increased markedly in numbers. The Federal Aviation Administration (then the Civil Aeronautics Authority) investigated many cases of flutter and noted that these could arise from maintenance problems such as the accumulation of ice or water internally, the painting of lifting surfaces, free play or backlash in controls, ineffective dampers, by loss of balance weights, or fatigue of balance weight arms. Practical guidelines for designers and operators have been given and updated in documents by the FAA (Federal Air Regulations), by the Air Force (Military Specifications), and by NASA (Space Vehicle Design Criteria).

The Computer Revolution and Finite Element Modeling

After the war, development of computing machines proceeded in two paths, analog and digital. The analog machines were mostly patterned after the Differential Analyzer of Vannevar Bush, which was essentially a passive mechanical means of solving linear differential equations with variable coefficients. At the California Institute of Technology, R.H. MacNeal, G. D. McCann, and C.H. Wilts[79] applied such methods electrically, including active elements to flutter analysis. Figure 16 is an illustration of the kind of results by analog methods which would have been prohibitively laborious by normal computational procedures then in use. It shows contours of flutter speed for a tapered wing including the effect of varying an engine mass location. Such results can explain the rationale for the particular placement of nacelles on transport aircraft. In England, F.

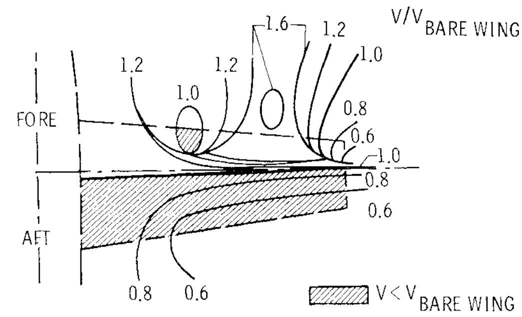

Fig. 16 Flutter speed contours of wing with attached mass obtained by electric analog solutions.

Smith used a six degree-of-freedom flutter simulator, and in France, L. Malavard used electrical analogs for flow solutions.

The revolution in digital computing machines that has transformed our world emerged slowly toward the end of the forties. One of the early machines used at Langley was a Bell computer using telephone relays, soon to be replaced by electronic types. John von Neumann, a pioneer of modern computing machines, was greatly influenced in sparking the modern development of computers by the extensive numerical effort required for treating shock waves and for predicting the weather. Flutter calculations, it seems, were a later influence. One of the authors recalls attending in the early fifties a symposium on flutter sponsored by the IBM Corporation. At lunch, he sat with Thomas B. Watson Jr. (recently ambassador to the U.S.S.R.) who stated that he was a pilot during the war and "knew about flutter." IBM, no doubt, sponsored the meeting because computing machines were increasingly being used by aircraft companies, and prominently for flutter analysis. Mathematical methods that had been considered academic, requiring prodigious numbers of man years of calculations by earlier methods, became feasible. A classical way of utilizing computers is to model the physical situation analytically by means of difference equations. A more recent type of mathematical modeling, representing a direct engineering approach and utilizing matrices, is now referred to as finite element analysis. It had its beginnings in the 1950s due to the work of many people largely in the fields of structures and aeroelasticity. In particular, in structures we may mention J.H. Argyris (see Survey Papers, 1966 and 1970) and a report by M. J. Turner and associates.[80] In aerodynamics, as implied earlier, numerical methods for lifting surfaces, for both steady and unsteady flows, employ discrete lattice, box, or panel methods, and have led to diverse methods of finite element analysis. At present, with computers capable of many millions of arithmetic operations per second, finite element analysis has become a dominant method in design.

The Transonic Dynamics Tunnel

The lack of suitable wind-tunnel facilities for determining the aeroelastic and flutter behavior of new high-speed aircraft designs influenced A. A. Regier to propose in 1951 that the NACA construct a large transonic wind tunnel dedicated to research and tests in the field of aeroelasticity. In justifying this significant proposal, Regier stated, "Present trends in designs of high performance aircraft lead to configurations and operating conditions which do not lend themselves to the theoretical treatment of flutter and associated dynamic problems. The speed ranges of interest are also such as to cast doubt on existing analyses. For these reasons designers are turning toward the use of dynamic models in order to determine aeroelastic and flutter behavior of proposed designs." The proposed tunnel facility was to have the following features: 1) be as large as feasible to enable accurate simulation of model details, such as control surfaces;

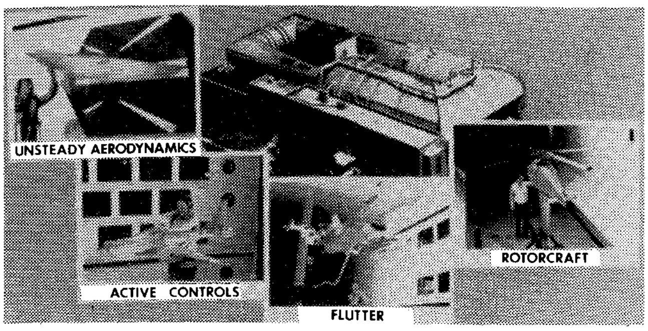

Fig. 17 Some technology areas supported by the Langley Transonic Dynamics Tunnel.

2) be capable of operating over a wide density range in order to simulate various altitude conditions, because flutter characteristics often change with altitude; 3) use Freon gas as the test medium which, based on previous experience, enables the use of heavier, less expensive models, permits higher Reynolds numbers, and requires less tunnel power; and 4) be capable of operating at Mach numbers up to 1.2.

This proposal was implemented, starting in 1955, by converting the Langley 19-ft Pressure Tunnel to a 16-ft (4.87-m) transonic tunnel with Freon-12 as a test medium. Designated the Transonic Dynamics Tunnel (TDT), the facility became operational in 1960 and has since been used almost exclusively to support research and development testing in the field of aeroelasticity. Figure 17 depicts the TDT and some of the important programs it supports. For example, the facility is used to verify, by means of dynamic models, the flutter safety and aeroelastic characteristics of most U.S. high-speed military aircraft and commercial transport designs; to explore flutter trends and aeroelastic characteristics of new configurations; for active control of aeroelastic response of airplanes and rotorcraft; for ground wind loads, flutter and buffet testing of space launch vehicles; and for unsteady aerodynamic load measurements on oscillating wings and control surfaces. Some wind-tunnel/flight correlations presented by Reed[81] indicate that predictions from aeroelastic models in the TDT were, in general, substantiated in flight.

Concluding Remarks

Although we have concentrated on the contributions of individuals, many organizations have contributed to the growth of knowledge in the areas of aeroelasticity and flutter. Among these are the RAE and NPL organizations in England, NLR in the Netherlands, ONERA in France, and DFVLR in Germany. In the United States there has been the U.S. Air Force Flight Dynamics Laboratory, the Navy Bureau of Aeronautics, the NACA and its Subcommittee on Vibration and Flutter, the Aerospace Flutter and Dynamics Council composed of industry specialists, the MIT Aeroelastic Laboratory, and the Cornell Aeronautical Laboratory. Special mention should be made of the NATO-AGARD organization, which has sponsored the six-volume *AGARD Manual on Aeroelasticity,* and many specialist symposia publications.

We leave the subject approaching its maturity in the mid-1950s. In closing, may we state that we are aware of the many shortcomings of this brief historical account. The Survey Papers should help furnish interested readers with other vistas of aeroelasticity and flutter and will supply the numerous individual references that could not be included. We may fully expect that, in time, the historical task will be done to the depth and breadth this intriguing subject merits.

Acknowledgments

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